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M.V. Temerbayeva<sup>1\*</sup>, M.N. Shkolnikova<sup>2</sup>

<sup>1</sup>Innovative University of Eurasia, Kazakhstan

<sup>2</sup>Biysk Institute of Technology (branch) of the Altai State Technical University  
named after I.I. Polzunov, Russia

\*(e-mail: marvik75@yandex.ru)

### **Mathematical modeling of the process of formation of the structure of bio-yoghurt based on goat's milk**

#### **Annotation**

*Main problem:* the article is devoted to the study of the degree of influence of physical and chemical factors on the formation of the structure of bio-yoghurt using mathematical modeling methods. The author determined the maximum allowable values of the Cochran criteria and the calculated value of Fisher, analyzed the coefficients of the regression equation. Controlled factors were selected: the number of stabilizing systems, temperature and duration of whipping. Controlled factors were selected: overrunning of a mixture of components, relative stability of overrunning of bio-yoghurt during storage, effective viscosity coefficient, organoleptic assessment of consistency

*Purpose:* obtaining objective data on the joint degree of influence of physical and chemical factors on the formation of the structure of bio-yoghurt from goat's milk with high organoleptic indicators, that is, consumer properties.

*Methods:* the article uses methods of mathematical analysis and mathematical-graphical analysis, mathematical statistics, matrix method.

*Results and their significance:* in experimental studies, the comparative effect of the stabilizing systems "GRINDSTED® SB 550A" and "CREMODAN SUPER" on the formation of the structure of bio-yoghurt based on goat's milk and their stability during storage was studied. The complex results of experimental studies are presented, characterizing the degree of influence of the type and amount of food additives on the consumer characteristics of bio-yoghurt based on goat's milk. The results of a comparative analysis of the data obtained as a result of the first two stages of mathematical modeling are presented: the type of food additive and its rational amount, as well as the parameters of the technological process - beating the product. 4% to 1.0% by weight of the formulation components in all whipping modes, an increase in the coefficient of dynamic viscosity of the product is observed, another indicator "overrun" decreases when the stabilizing system "GRINDSTED® SB 550A" is added in an amount of 1.0%.

*Keywords:* mathematical modeling, food additive, controlled factors, controlled factors, objective function, stabilizing system.

#### **Introduction**

Bio-yoghurt based on goat's milk is a complex multicomponent system, which includes food products of various structuring: liquid, dry, concentrated. When forming a structured fermented product, it should be taken into account that food products are complex water-polymer systems, which include proteins, carbohydrates, lipids, minerals, the ratio of which determines the consumer properties of the product. Technological properties, including the consistency of food products, depend on the degree of interaction between all the constituent components. They can be changed under the influence of various external factors:

- energy field (steam, hot air, high pressure, infrared radiation);
- physico-chemical (mixing, whipping, rubbing) with the addition of special additives - chemicals of natural or artificial origin.

Bio-yoghurt based on goat's milk is a complex biosystem. To give it high consumer properties, including a light whipped structure, at this stage it is necessary to use physical and chemical factors: whipping and stabilizing systems. Given the above, the topic of this scientific study is relevant and timely.

### Materials and methods

The main goal of this stage of experimental and analytical research is to obtain objective data on the joint degree of influence of physicochemical factors on the formation of the structure of bio-yoghurt based on goat's milk with high organoleptic indicators, that is, consumer properties.

Initially, experimental data, in order to identify and eliminate gross experimental errors (misses), were processed by methods of mathematical statistics. To do this, the homogeneity of variance estimates was determined using the Cochran test according to the following dependence:

$$G_p = \frac{\max S_i^2}{\sum_{i=1}^N S_i^2}, \quad (1)$$

где:  $\max S_i^2$  - maximum variance of a series of parallel experiments;

$$\sum_{i=1}^N S_i^2 - \text{sum of variances.}$$

For each series of parallel experiments, the arithmetic mean of the response function was calculated:

$$Y_{cp} = \frac{1}{k} \sum_{i=1}^k Y_{3,i}, \quad (2)$$

The variance of the experiments was calculated by the formula:

$$S_i^2 = \frac{1}{k-1} \sum_{i=1}^k (Y_{3,i} - Y_{cp})^2, \quad (3)$$

where  $i$  – is the serial number;

$k$  – the number of parallel experiments conducted under the same conditions;

$N$  – the number of series of experiments (the number of estimates of variances).

The hypothesis of uniformity of variances was accepted if the calculated value of the Cochran criterion  $G_p$  is less than the tabular  $G_{table}$ , that is,  $G_p < G_{table}$ .

Critical, that is, the maximum permissible values of the Kohren criteria were determined by tables. To find the Cochran criterion  $G$ , it is necessary to know the total number  $N$  of variance estimates and the number of degrees of freedom  $f$  associated with each of them, and  $f = k - 1$ . Experiments are considered reproducible when the condition  $G_p \leq G_{table}$  is met.

If the experiments are reproducible, then the calculation of the variance of reproducibility of a series of parallel experiments is calculated by the formula:

$$S_{\text{восп}}^2 = \frac{1}{N} \sum_{i=1}^N S_i^2, \quad (4)$$

In this case, the number of degrees of freedom is determined by the formula [1]:

$$f = N(k-1), \quad (5)$$

At the second stage of mathematical processing, the coefficients of the  $b_i$  regression equation were determined.

The relationship between the response function  $Y$  and the selected factors in the study area is linear. In this regard, we are looking for this dependence in the form of a linear regression equation [2]:



$$S_b^2 = \frac{S_y^2}{N}, \quad (12)$$

It is known that the regression coefficient is significant if the condition is met:

$$|b| \geq S_b \cdot t, \quad (13)$$

Where  $t$  - the value of the Student's criterion is determined from the table for a given reliability of the experiment ( $p$ ) and the number of degrees of freedom ( $f$ ). Moreover, the number of degrees of freedom is equal to:

$$f = N(k - 1), \quad (14)$$

where  $N$  - number of experiments;

$k$  - the repetition rate of each experiment (for our case  $k = 5$ ).

Having received the regression equation, we check its adequacy, i.e. the ability to describe the response surface well enough and predict the results of experiments. To check the adequacy, an estimate of the adequacy variance was calculated using the formula [4]:

$$S_{ад}^2 = \frac{1}{N - B} \sum_{i=1}^N (y_{эi} - y_{pi})^2, \quad (15)$$

where  $B$  - number of significant regression coefficients;

$y_{эi}, y_{pi}$  - experimental and calculated value of the response function in the experiment;

$N$  - the number of experiments of the full factorial experiment.

The number of degrees of freedom is associated with the estimation of the adequacy variance  $f = N - B$ .

The next processing step was to determine the calculated value of the Fisher criterion [5]:

$$F_p = \frac{S_{ад}^2}{S_y^2}, \quad (16)$$

The regression equation is considered adequate if the condition is met:

$$F_p \leq F, \quad (17)$$

where  $F$  – the critical (tabular) value of the Fisher criterion.

To check the adequacy of the regression equation, we also used the correlation ratio, which was calculated using the formula:

$$\eta = \sqrt{1 - \frac{\sum_{i=1}^n (Y_{э,i} - Y_{p,i})^2}{\sum_{i=1}^n (Y_{э,i} - Y_{cp})^2}}, \quad (18)$$

The average relative prediction error (%) was calculated using the formula:

$$E = \frac{100}{n} \sum_{i=1}^n \left| \frac{Y_{э,i} - Y_{p,i}}{Y_{э,i}} \right|, \quad (19)$$

where  $Y_{\Sigma,i}$ ,  $Y_{p,i}$  и  $Y_{cp}$  - experimental, calculated and average values of the Y;  
 n - the number of experimental Y values used to verify adequacy.

### Results

The research was carried out in stages and consisted of a mathematical and graphical analysis of experimental data from numerous series of experiments conducted in threefold repetition:

The main task is to conduct a mathematical analysis and modeling of the process of whipping the mass of components of bio-yogurt with the stabilizing system "GRINDSTED® SB 550A" and CREMODAN SUPER, to establish how many of them and under what conditions of whipping allows to obtain a product with improved consumer characteristics. To conduct a comparative analysis of the data obtained as a result of the first two stages of mathematical modeling and as a result to determine: the type of food additive and its rational amount; the parameters of the technological process – whipping.

The regulated factors of this study are the following:

$X_1$  – quantity «GRINDSTED® SB 550A», %;

$X_2$  – quantity CREMODAN SUPER, %;

$X_3$  – whipping temperature, °C;

$X_4$  – duration of whipping, minutes.

Controlled factors are selected:

$Y_1$  – the whipping of the mixture of components is  $\rightarrow$  max, the limit is no more than 40 %;

$Y_2$  – the relative stability of the whipping of the bio-yogurt during storage is  $\rightarrow$  100 %, the limit is not less than 80 %;

$Y_3$  – coefficient of effective viscosity no more than 1.4 MPa·s;

$Y_4$  – organoleptic consistency score,  $\rightarrow$  10 points, a limit of at least 8.0 points.

The comprehensive results of experimental studies characterizing the degree of influence of the type and quantity of food additives on the consumer characteristics of bio-yoghurt are shown in Tables 1 and 2.

Table 1 – Analysis of regulated factors in the formation of the bio-yoghurt structure based on goat's milk

Product	Regulated factors			
	Food additive, %		$X_3, ^\circ\text{C}$	$X_4$ , minutes
	$X_1$	$X_2$		
Experience 1	0,4	–	30	6
Experience 2	0,6	–	30	6
Experience 3	0,8	–	30	6
Experience 4	1,0	–	30	6
Experience 5	0,4	–	30	9
Experience 6	0,6	–	30	9
Experience 7	0,8	–	30	9
Experience 8	1,0	–	30	9
Experience 9	0,4	–	30	12
Experience 10	0,6	–	30	12
Experience 11	0,8	–	30	12
Experience 12	1,0	–	30	12
Experience 13	0,4	–	10	6
Experience 14	0,6	–	10	6
Experience 15	0,8	–	10	6
Experience 16	1,0	–	10	6
Experience 17	0,4	–	10	9
Experience 18	0,6	–	10	9
Experience 19	0,8	–	10	9
Experience 20	1,0	–	10	9

Table 2 – Analysis of controlled factors in the formation of the bio-yoghurt structure based on goat's milk

Product	Controlled factors			
	Y <sub>1</sub> , %	Y <sub>2</sub> , %	Y <sub>3</sub> , мПа·с	Y <sub>4</sub> , score
Experience 1	16,0	63,75	1,10	6,0
Experience 2	20,0	78,40	1,32	6,9
Experience 3	26,0	81,53	2,91	7,5
Experience 4	18,0	72,22	3,55	6,0
Experience 5	18,0	72,36	1,15	6,5
Experience 6	24,0	82,33	1,34	7,5
Experience 7	35,2	82,78	3,00	8,0
Experience 8	19,0	76,35	3,60	6,5
Experience 9	20,0	75,00	1,20	7,0
Experience 10	28,0	83,64	1,40	8,3
Experience 11	42,0	84,03	3,20	9,0
Experience 12	20,0	77,08	3,75	6,8
Experience 13	24,8	72,76	1,15	7,5
Experience 14	29,0	78,52	1,22	8,9
Experience 15	42,0	78,47	2,18	9,7
Experience 16	15,2	67,73	3,10	6,5
Experience 17	28,2	75,58	1,16	7,8
Experience 18	30,5	81,89	1,31	8,0
Experience 19	44,6	83,53	2,27	9,4
Experience 20	19,2	72,50	3,22	7,0

The translation of controlled factors into dimensionless quantities for mathematical modeling of complex experimental results is presented in Table 3.

Table 3 – Analysis of regulated and controlled factors when translated into dimensionless quantities

Product	Regulated factors				Controlled factors			
	food additive, %		X <sub>3</sub> , °C	X <sub>4</sub> , мин	Y <sub>1</sub> , %	Y <sub>2</sub> , %	Y <sub>3</sub> , мПа·с	Y <sub>4</sub> , score
	X <sub>1</sub>	X <sub>2</sub>						
Experience 1	0,4	–	30	6	16,0	63,75	1,10	6,0
Experience 2	0,6	–	30	6	20,0	78,40	1,32	6,9
Experience 3	0,8	–	30	6	26,0	81,53	2,91	7,5
Experience 4	1,0	–	30	6	18,0	72,22	3,55	6,0
Experience 5	0,4	–	30	9	18,0	72,36	1,15	6,5
Experience 6	0,6	–	30	9	24,0	82,33	1,34	7,5
Experience 7	0,8	–	30	9	35,2	82,78	3,00	8,0
Experience 8	1,0	–	30	9	19,0	76,35	3,60	6,5
Experience 9	0,4	–	30	12	20,0	75,00	1,20	7,0
Experience 10	0,6	–	30	12	28,0	83,64	1,40	8,3
Experience 11	0,8	–	30	12	42,0	84,03	3,20	9,0
Experience 12	1,0	–	30	12	20,0	77,08	3,75	6,8
Experience 13	0,4	–	10	6	24,8	72,76	1,15	7,5
Experience 14	0,6	–	10	6	29,0	78,52	1,22	8,9
Experience 15	0,8	–	10	6	42,0	78,47	2,18	9,7
Experience 16	1,0	–	10	6	15,2	67,73	3,10	6,5
Experience 17	0,4	–	10	9	28,2	75,58	1,16	7,8
Experience 18	0,6	–	10	9	30,5	81,89	1,31	8,0
Experience 19	0,8	–	10	9	44,6	83,53	2,27	9,4
Experience 20	1,0	–	10	9	19,2	72,50	3,22	7,0

To solve further problems, we have translated the controlled factors into dimensionless quantities using the coefficient of significance of indicators. The calculation of the objective function  $\sum_{n=0}^5 Y_0$  was performed for each indicator. The results are shown in table 4.

Table 4 – Results of calculation of the objective function

Product	Translation of controlled factors into dimensionless quantities				$\sum_{n=0}^5 Y_0$
	$Y_1$	$Y_2$	$Y_3$	$Y_4$	
Experience 1	0,40	0,00	0,79	0,00	1,19
Experience 2	0,50	0,00	0,94	0,00	1,44
Experience 3	0,65	0,96	0,00	0,00	1,61
Experience 4	0,45	0,00	0,00	0,00	0,45
Experience 5	0,45	0,00	0,82	0,00	1,27
Experience 6	0,60	0,97	0,96	0,00	2,52
Experience 7	0,88	0,97	0,00	0,80	2,65
Experience 8	0,48	0,00	0,00	0,00	0,48
Experience 9	0,50	0,00	0,86	0,00	1,36
Experience 10	0,70	0,98	1,00	0,83	3,51
Experience 11	0,00	0,99	0,00	0,90	1,89
Experience 12	0,50	0,00	0,00	0,00	0,50
Experience 13	0,62	0,00	0,82	0,00	1,44
Experience 14	0,73	0,00	0,87	0,89	2,49
Experience 15	0,00	0,00	0,00	0,97	0,97
Experience 16	0,38	0,00	0,00	0,00	0,38
Experience 17	0,71	0,00	0,83	0,00	1,53
Experience 18	0,76	0,96	0,94	0,80	3,46
Experience 19	0,00	0,98	0,00	0,94	1,92
Experience 20	0,48	0,00	0,00	0,00	0,48

### Discussion

The analysis of the data obtained made it possible to solve the main task set in this section – the choice of optimal whipping conditions and a stabilizer that allows getting a sour-milk dessert with high organoleptic characteristics and a stable structure in storage.

### Conclusion

Characterizing the results of experimental studies, it can be noted that as the amount of the stabilizing system increased from 0.4% to 1.0% of the mass of the components of the formulation, an increase in the dynamic viscosity coefficient was observed in all whipping modes, another indicator "whipping" decreased when the GRINDSTED® SB 550A stabilizing system was introduced in an amount of 1.0%. A significant effect on the foaming process was influenced by both the temperature and the whipping time. The stability of the foam during product storage was also evaluated.

Considering the results of the use of another food additive – CREMODAN SUPER, it can be noted that its use in the technology of bio-yoghurt with a whipped structure was characterized by lower efficiency, that is, the whipping of products was less. The foam was characterized by instability in storage.

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**М.В. Темербаева<sup>1\*</sup>, М.Н. Школьников<sup>2</sup>**

<sup>1</sup>Инновациялық Еуразия университеті, Қазақстан

<sup>2</sup> «Алтай мемлекеттік техникалық университеті» ФГБОУ Бийск технологиялық институты (филиалы). И. И. Ползунова", Ресей

### **Ешкі сүтіне негізделген биогурт құрылымын қалыптастыру үрдісін математикалық модельдеу**

Мақала математикалық модельдеу әдістерін қолдана отырып, биогурт құрылымын қалыптастыруға физикалық-химиялық факторлардың әсер ету дәрежесін зерттеуге арналған. Автор Кохрен критерийлерінің шекті рұқсат етілген мәндерін және Фишердің есептік мәнін анықтады, регрессия теңдеуінің коэффициенттерін талдады. Зертханалық және өндірістік жағдайларда алынған эксперименттік-аналитикалық материал негізінде математикалық модельдеу жүргізілді. Реттелетін факторлар таңдалады (тұрақтандыру жүйелерінің саны, температура және қамшының ұзақтығы. Басқарылатын факторлар таңдалады (компоненттер қоспасының шайқалуы, сақтау үрдісінде биогурттың шайқалуының салыстырмалы тұрақтылығы, тиімді тұтқырлық коэффициенті, консистенцияны органолептикалық бағалау). "GRINDSTED® SB 550A" және "CREMODAN SUPER" тұрақтандырушы жүйелерімен биогурт компоненттерінің массасын шайқау үрдісіне Математикалық талдау және модельдеу жүргізілді.

Эксперименттік зерттеулерде "GRINDSTED® SB 550A" және "CREMODAN SUPER" тұрақтандырушы жүйелерінің ешкі сүтіне негізделген био-гурт құрылымын қалыптастыруға және олардың сақтау тұрақтылығына салыстырмалы әсері зерттелді. Ешкі сүтіне негізделген биогурттың тұтынушылық сипаттамаларына тағамдық қоспалардың түрі мен мөлшерінің әсер ету дәрежесін сипаттайтын эксперименттік зерттеулердің кешенді нәтижелері ұсынылған. Математикалық модельдеудің алғашқы екі кезеңінен алынған деректерді салыстырмалы талдау нәтижелері келтірілген: тағамдық қоспаның түрі және оның ұтымды мөлшері, сондай-ақ технологиялық үрдістің параметрлері - өнімді шайқау. "GRINDSTED® SB 550A" тұрақтандырғыш жүйесі мөлшерінің барлық шайқау режимдерінде формула компоненттері массасының 0,4% – дан 1,0% - на дейін ұлғаю шамасында өнімнің динамикалық тұтқырлығы коэффициентінің ұлғаюы байқалатыны дәлелденді, "шайқалудың" басқа көрсеткіші - "grindsted® SB 550A" тұрақтандырғыш жүйесін 1,0% мөлшерінде енгізген кезде азаяды.



Түйін сөздер: математикалық модельдеу, тағамдық қоспалар, басқарылатын факторлар, басқарылатын факторлар, мақсатты функция, тұрақтандырушы жүйе.

**М.В. Темербаева<sup>1\*</sup>, М.Н. Школьников<sup>2</sup>**

<sup>1</sup>Инновациялық Еуразия университеті, Қазақстан

<sup>2</sup>Бийский технологический институт (филиал) ФГБОУ ВО «Алтайский государственный технический университет им. И.И. Ползунова», Россия

### **Математическое моделирование процесса формирования структуры биойогурта на основе козьего молока**

Статья посвящена исследованию влияния физико-химических факторов на формирование структуры биойогурта с применением методов математического моделирования. Автором определены предельно допустимые значения критериев Кохрена и расчетное значение Фишера, проанализированы коэффициенты уравнения регрессии. Проведено математическое моделирование на базе экспериментально-аналитического материала, полученного в лабораторных и производственных условиях. Выбраны регулируемые факторы (количество стабилизирующих систем, температура и продолжительность взбивания), управляемые факторы (взбитость смеси компонентов, относительная устойчивость взбитости биойогурта в процессе хранения, коэффициент эффективной вязкости, органолептическая оценка консистенции). Проведен математический анализ и моделирование процесса взбивания массы компонентов биойогурта со стабилизирующими системами «GRINDSTED® SB 550A» и «CREMODAN SUPER» с установлением их количества и условиями взбивания.

В экспериментальных исследованиях было изучено сравнительное влияние стабилизирующих систем «GRINDSTED® SB 550A» и «CREMODAN SUPER» на формирование структуры биойогурта на основе козьего молока и их устойчивость при хранении. Представлены комплексные результаты, характеризующие степень влияния вида и количества пищевых добавок на потребительские характеристики биойогурта на основе козьего молока. Приведены результаты сравнительного анализа данных, полученных в результате проведения двух первых этапов математического моделирования: вид пищевой добавки и её рациональное количество, а также параметры технологического процесса – взбивания продукта. Доказано, что мере увеличения количества стабилизирующей системы «GRINDSTED® SB 550A» с 0,4 % до 1,0 % от массы компонентов рецептуры при всех режимах взбивания наблюдается увеличение коэффициента динамической вязкости продукта, другой показатель «взбитость» – снижается при внесении стабилизирующей системы «GRINDSTED® SB 550A» в количестве 1,0 %.

Ключевые слова: математическое моделирование, пищевая добавка, управляемые факторы, стабилизирующая система.

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